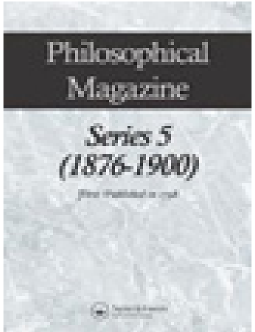

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LX. *On the Application of* Sir William Thomson's *Theory of a Contractile Æther to Double Refraction, Dispersion, Metallic Reflexion, and other Optical Problems. By* R. T. Glazebrook, *M.A., F.R.S.**

IN any isotropic elastic solid there are, in general, two velocities of wave-propagation—one for normal waves, given by $\sqrt{A/\rho}$ in Green's notation; the other, given by $\sqrt{B/\rho}$, for transverse waves; and when any system of waves falls on the bounding surface of two such media both these disturbances are set up. Since light-waves are entirely transverse, and do not give rise to normal waves possessing, at any rate, more than a very small fraction of the energy of the incident waves, it follows, as was shown by Green, that the ratio A/B is, for the æther, either extremely large or extremely small. If the surfaces of the solid at a finite distance from the origin be free, it is necessary, in order that the equilibrium position may be one of minimum potential energy, that $A-\frac{4}{3}B$ should be positive, and hence Green supposed that A was very large and the æther incompressible. This view has generally been accepted by English writers on optical subjects.

In his paper "On the Reflexion and Refraction of Light," in the last number of this Magazine, Sir William Thomson, however, has shown that, "*provided we suppose the medium to extend all through boundless space or give it a fixed containing vessel as its boundary,*" the conditions for stability in the æther are satisfied if we suppose that neither A nor B is negative. Under these circumstances it is not necessary that A should be greater than $\frac{4}{3}B$, it is sufficient that A should be zero or positive. Such a medium, according to Sir William Thomson, is afforded us by homogeneous airless foam held from collapse by adhesion to a bounding vessel which may be infinitely distant all round, and for this medium A is zero, *i. e.* the medium is incapable of transmitting normal waves. On this hypothesis as to the nature of the æther it is possible to suppose that the absence of the normal wave is because A is zero, not because it is infinite. Sir William Thomson has, in in his paper just referred to, treated the problem of reflexion and refraction on this supposition; the object of the present communication is to consider double refraction and other allied problems.

In my Report on Optical Theories, presented to the British Association at Aberdeen (B. A. Report, 1885, p. 179), when discussing the equations which are given by certain theories

* Communicated by the Author.

of double refraction, I say :—"The question arises, Are these equations incompatible with Fresnel's wave-surface? Lord Rayleigh has of course proved that they are if the equation

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=0,$$

expresses an absolutely necessary condition;" *i. e.* if the æther is incompressible, u, v, w being the displacements; "but it is not difficult to show that if, instead of the above equation, we put

$$\frac{1}{a^2}\frac{du}{dx}+\frac{1}{b^2}\frac{dv}{dy}+\frac{1}{c^2}\frac{dw}{dz}=0$$

(a, b, c being the principal wave-velocities), then the wave-surface will be Fresnel's, the direction of vibration will be normal to the ray, and will be in a plane containing the ray, the wave-normal, and an axis of the section of the ellipsoid $a^2x^2+b^2y^2+c^2z^2=1$ by the wave-front, while the velocity of propagation will be inversely proportional to the length of this axis."

At the date at which this extract was written I believed that the æther must necessarily be incompressible, and therefore that the suggestion there made was impossible. The recent paper of Sir William Thomson's has shown that the condition of incompressibility is not necessary, and I propose, therefore, to develop the theory given in outline in the Report.

Before so doing I wish to refer to three points in Sir William Thomson's paper. He shows there that, under the conditions already stated, viz. no motion at infinity, the expression

$$\begin{aligned}W=\tfrac{1}{2}\iiint dx\,dy\,dz\,\Big[&A\Big(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\Big)^2\\&+B\Big\{\Big(\frac{dw}{dy}+\frac{dv}{dz}\Big)^2+\Big(\frac{du}{dz}+\frac{dw}{dx}\Big)^2+\Big(\frac{dv}{dx}+\frac{du}{dy}\Big)^2\Big\}\\&-4B\Big\{\frac{dv}{dy}\frac{dw}{dz}+\frac{dw}{dz}\frac{du}{dx}+\frac{du}{dx}\frac{dv}{dy}\Big\}\Big],\quad\ldots\ldots\quad(1)\end{aligned}$$

which is Green's value for the work required to strain the solid, transforms into

$$\begin{aligned}\tfrac{1}{2}\iiint dx\,dy\,dz\,\Big[&A\Big(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\Big)^2\\&+B\Big\{\Big(\frac{dw}{dy}-\frac{dv}{dz}\Big)^2+\Big(\frac{du}{dz}-\frac{dw}{dx}\Big)^2+\Big(\frac{dv}{dx}-\frac{du}{dy}\Big)^2\Big\}\Big],\quad .\quad(2)\end{aligned}$$

and then, for the optical problem, A is put equal to zero. Now the term with B for a coefficient in this expression is exactly the expression assumed by MacCullagh in his theory of reflexion, and which Stokes has shown (B. A. Report, 1862, p. 278) to be impossible as an expression for the energy of a strained solid, for it leads to the equations $T_{xy} = -T_{yx}$, &c., instead of $T_{xy} = T_{yx}$, where T_{xy} means the stress parallel to y on a plane normal to x; how, then, can it represent the energy of the strained medium?

The explanation of this point is simple. The second expression for W only gives the energy of the *whole* solid under certain surface-conditions. Each element of the integral is not an expression for the energy of the corresponding element of the solid; to find this we have to take into account the surface-integrals introduced by the transformation. These surface-integrals it is true vanish when the whole medium is considered; but in calculating the stresses on each element they are of importance, and when they are taken into account the true values are found for T_{xy} &c. We cannot get these values from the transformed expression directly, for that is only true under certain conditions.

A second point is the following:—The integral

$$\iiint\left(\frac{dv}{dy}\frac{dw}{dz}+\ldots\right)dx\,dy\,dz$$

is transformed into

$$\iiint\left(\frac{dv}{dz}\frac{dw}{dy}+\ldots\right)dx\,dy\,dz$$

+ certain surface-integrals.

These surface-integrals vanish if u, v, w are all zero at the surface. *They also vanish whenever u, v, w are functions of the same function of x, y, z and t.* Thus, as I pointed out in a paper on the Reflexion and Refraction of Light (Proc. Camb. Phil. Soc. vol. iii. 1880), if W denote the true expression for the work W′, the transformed expression W = W′ + M, where M is a quantity which may be negative, but which vanishes if u, v, w are functions of the same function of the variables.

There remains the third point. Let us suppose that, in transforming, as is done by Sir William Thomson, the integral for W we pass across a surface at a finite distance from the origin, in crossing which the rigidity changes from B to B′. Unless either there is no motion over this surface, which is impossible, or certain relations hold in addition to the ordinary surface-conditions among the stresses, implying of course the existence of surface-tractions &c. other than those which arise from the strains, the surface-integral occurring in the transformation does not vanish, and the surface contributes something to the energy. It follows, hence, that we must have the condition $B=B'$ satisfied, and optical differences must arise from differences in the optical density of the æther on either side of the surface. In the paper this relation is assumed to simplify the formulæ, the foregoing considerations show that it is necessary. This point, however, is dealt with by Sir William Thomson himself in a note in this number of the Philosophical Magazine.

To turn now to the problem of Double Refraction. Since, according to our theory, the rigidity of the æther is the same in all media, it is clear that we cannot explain double refraction by variation of rigidity in different directions in a crystal, and we are driven to consider the hypothesis advanced by Rankine, Stokes, and Rayleigh, and which has been shown by the latter two to lead, if the æther be incompressible, to a wave-surface other than that of Fresnel.

According to this hypothesis the density of the æther is to be treated as a function of the direction of displacement. The kinetic energy will be a quadratic function of the displacements, and for one set of axes may be written

$$\tfrac{1}{2}\iiint (\rho_x \dot{u}^2 + \rho_y \dot{v}^2 + \rho_z \dot{w}^2)\, dx\, dy\, dz.$$

We suppose that these axes coincide with the axes of the crystal. The potential energy has, according to our supposition, its usual form, and the constants A, B are the same as those for an isotropic medium. Thus, following Lord Rayleigh's paper (Phil. Mag. June 1871), we have as the equations of motion* :—

$$\left.\begin{aligned} \rho_x \frac{d^2u}{dt^2} &= (A-B)\frac{d\delta}{dx} + B\nabla^2 u \\ \rho_y \frac{d^2v}{dt^2} &= (A-B)\frac{d\delta}{dy} + B\nabla^2 v \\ \rho_z \frac{d^2w}{dt^2} &= (A-B)\frac{d\delta}{dz} + B\nabla^2 w \end{aligned}\right\} \quad . \quad . \quad . \quad (3)$$

where $$\delta = \frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence, differentiating with regard to x, y, z, and adding,

$$\frac{d^2}{dt^2}\left(\rho_x \frac{du}{dx} + \rho_y \frac{dv}{dy} + \rho_z \frac{dw}{dz}\right) = A\nabla^2\delta. \quad . \quad . \quad . \quad (5)$$

* A theory leading to equations practically the same as these has been given by Sarrau, *Liouville Journal*, ser. ii. tomes xii. and xiii. (see Glazebrook, "Report on Optical Theories," B. A. Report, 1885, p. 174), and by Boussinesq, *Liouville Journal*, ser. ii. tome xiii. (Glazebrook, "Report on Optical Theories," p. 213), and in these the same conclusions as to the direction of vibration are arrived at.

Let $lx+my+nz-Vt$ give the position of the wave-front at any instant. Let λ, μ, ν, be the direction-cosines of the displacement, and let Θ be the amount of the displacement. Then

$$u=\lambda\Theta, \quad v=\mu\Theta, \quad w=\nu\Theta,$$

and hence

$$\lambda\rho_x\frac{d^2\Theta}{dt^2}=(A-B)\frac{d}{dx}\left(\lambda\frac{d\Theta}{dx}+\mu\frac{d\Theta}{dy}+\nu\frac{d\Theta}{dz}\right)+B\lambda\nabla^2\Theta, \quad (6)$$

&c. . . . .

Now let

$$\Theta=\Theta_0 e^{\iota(lx+my+nz-Vt)}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

then

$$\delta=\Theta\iota(l\lambda+m\mu+n\nu). \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Substituting in the equations we find

$$\left.\begin{aligned}\lambda\rho_x V^2&=(A-B)l(l\lambda+m\mu+n\nu)+B\lambda\\ \mu\rho_y V^2&=(A-B)m(l\lambda+m\mu+n\nu)+B\mu\\ \nu\rho_z V^2&=(A-B)n(l\lambda+m\mu+n\nu)+B\nu\end{aligned}\right\}, \quad . \quad (9)$$

$$V^2\{\rho_x l\lambda+\rho_y m\mu+\rho_z n\nu\}=A(l\lambda+m\mu+n\nu). \quad . \quad (10)$$

Now put

$$a^2=B/\rho_x,\ b^2=B/\rho_y,\ c^2=B/\rho_z.$$

Then

$$\left.\begin{aligned}B\lambda\left(\frac{V^2}{a^2}-1\right)&=(A-B)l(l\lambda+m\mu+n\nu)\\ B\mu\left(\frac{V^2}{b^2}-1\right)&=(A-B)m(l\lambda+m\mu+n\nu)\\ B\nu\left(\frac{V^2}{c^2}-1\right)&=(A-B)n(l\lambda+m\mu+n\nu)\end{aligned}\right\}, \quad . \quad (11)$$

and

$$BV^2\left\{\frac{l\lambda}{a^2}+\frac{m\mu}{b^2}+\frac{n\nu}{c^2}\right\}=A(l\lambda+m\mu+n\nu). \quad . \quad . \quad (12)$$

Multiply the first of equations (11) by l, divide by (V^2-a^2), and so on, and add the three. Then we find

$$B\left(\frac{l\lambda}{a^2}+\frac{m\mu}{b^2}+\frac{n\nu}{c^2}\right)$$

$$=(A-B)(l\lambda+m\mu+n\nu)\left(\frac{l^2}{V^2-a^2}+\frac{m^2}{V^2-b^2}+\frac{n^2}{V^2-c^2}\right). \quad . \quad (13)$$

Hence, and from (12),

$$\frac{l^2}{V^2-a^2}+\frac{m^2}{V^2-b^2}+\frac{n^2}{V^2-c^2}=\frac{A}{A-B}\frac{1}{V^2}. \quad . \quad (14)$$

This is the general form of the equation of wave-slowness, without any assumption as to the relative magnitudes of A and B. If the æther be incompressible (Rankine, Stokes, Rayleigh), A is infinite, and the right-hand side is $1/V^2$. If, on the other hand, A vanishes (Thomson), the right-hand side is zero, and the equation becomes

$$\frac{l^2}{V^2-a^2}+\frac{m^2}{V^2-b^2}+\frac{n^2}{V^2-c^2}=0. \quad . \quad . \quad . \quad (15)$$

which is Fresnel's surface.

To trace the form of the surface in general, let us find its section by the principal planes. Suppose a, b, c to be in order of magnitude, and let $A/(A-B)=-k$. Consider the section by the plane of zx. Then $m=0$, and we have

$$(V^2-b^2)[V^2\{l^2(V^2-c^2)+n^2(V^2-a^2)\}+k(V^2-c^2)(V^2-a^2)]=0. \quad (16)$$

Thus the section consists of a circle given by $V=b$, and the quartic curve

$$V^4(1+k)-V^2\{(l^2c^2+n^2a^2)+k(a^2+c^2)\}+ka^2c^2=0. \quad . \quad (17)$$

The two important cases are given by $k=-1$ (Rankine, Stokes, Rayleigh); and k very small, probably zero (Thomson). For the latter case, on solving the quadratic and neglecting k^2 and higher powers, the two roots are

$$V_2{}^2=a^2n^2+c^2l^2+\frac{k(a^2-c^2)^2l^2n^2}{a^2n^2+c^2l^2}; \quad . \quad . \quad . \quad (18)$$

and

$$V_3{}^2(1+k)=\frac{ka^2c^2}{a^2n^2+c^2l^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Thus the section of the surface of wave-slowness by this plane will be, for the nearly transverse waves, the circle given by

$$\frac{1}{r^2}=b^2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

and a curve differing from an ellipse by extremely small quantities depending on $k(a^2-c^2)^2$, and given by

$$\frac{1}{r^2}=a^2n^2+c^2l^2+\frac{k(a^2-c^2)^2l^2n^2}{a^2n^2+c^2l^2}; \quad . \quad . \quad . \quad (21)$$

and for the condensational wave, the inverse of an ellipse, given by

$$\frac{1+k}{r^2}=\frac{ka^2c^2}{a^2n^2+c^2l^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Moreover for the velocity of the condensational wave along the axis of x we find the value $\sqrt{\frac{k}{1+k}}\,a$. If we substitute the values of a and k, this reduces to

$$\sqrt{A/\rho_x}.$$

The three principal normal velocities then are :—

$$\sqrt{A/\rho_x}, \quad \sqrt{A/\rho_y}, \quad \sqrt{A/\rho_z};$$

while for the nearly transverse waves they are:—

$$\sqrt{B/\rho_x}, \quad \sqrt{B/\rho_y}, \quad \sqrt{B/\rho_z}.$$

The sole condition, therefore, for the disappearance of the normal wave is that A should be extremely small compared with B. This is the same as for an isotropic medium.

Taking, now, the extreme case in which A vanishes in comparison with B; let us determine the relations between the direction of vibration λ, μ, ν, and that of propagation l, m, n.

If we put $A=0$ in (5), we get

$$\rho_x \frac{du}{dx} + \rho_y \frac{dv}{dy} + \rho_z \frac{dw}{dz} = 0, \quad . \quad . \quad . \quad . \quad (23)$$

or

$$\frac{l\lambda}{a^2} + \frac{m\mu}{b^2} + \frac{n\nu}{c^2} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Now if l, m, n are the direction-cosines of a normal to the ellipsoid,

$$a^2x^2 + b^2y^2 + c^2z^2 = 1, \quad . \quad . \quad . \quad . \quad (25)$$

then the direction-cosines of the line, joining the centre of the ellipsoid to the point of contact of the tangent-plane

$$lx + my + nz = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

are proportional to

$$l/a^2, \quad m/b^2, \quad n/c^2.$$

Hence λ, μ, ν, or the direction of vibration, lies in a plane normal to that radius vector of the ellipsoid (25) which is drawn to the point of contact of (26).

Again, from (9) we have

$$V^2 - a^2 = \frac{A-B}{B}(l\lambda + m\mu + n\nu)\frac{a^2 l}{\lambda}, \quad . \quad . \quad (27)$$

and two similar equations. Whence

$$\left.\begin{aligned} b^2 - a^2 &= \frac{A-B}{B}(l\lambda + m\mu + n\nu)\left(\frac{a^2 l}{\lambda} - \frac{b^2 m}{\mu}\right), \\ c^2 - b^2 &= \frac{A-B}{B}(l\lambda + m\mu + n\nu)\left(\frac{b^2 m}{\mu} - \frac{c^2 n}{\nu}\right); \end{aligned}\right\} \quad . \quad (28)$$

and from these,

$$\frac{a^2 l}{\lambda}(b^2 - c^2) + \frac{b^2 m}{\mu}(c^2 - a^2) + \frac{c^2 n}{\nu}(a^2 - b^2) = 0. \quad . \quad (29)$$

Thus, for all values of A and B, the line λ, μ, ν lies on the cone given by (29), and this with equation (24) determines its position completely.

Again, let x, y, z be the coordinates of the point in which the wave-front touches the wave-surface. Then we have*, if $x^2+y^2+z^2=r^2$,

$$lx+my+nz=\mathrm{V}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

$$\left.\begin{aligned} x&=l\mathrm{V}\frac{r^2-a^2}{\mathrm{V}^2-a^2}, \\ y&=m\mathrm{V}\frac{r^2-b^2}{\mathrm{V}^2-b^2}, \\ z&=n\mathrm{V}\frac{r^2-c^2}{\mathrm{V}^2-c^2}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Also from (9) or (27), we find

$$\frac{\lambda}{\frac{a^2l}{\mathrm{V}^2-a^2}} = \frac{\mu}{\frac{b^2m}{\mathrm{V}^2-b^2}} = \frac{\nu}{\frac{c^2n}{\mathrm{V}^2-c^2}} = \kappa \text{ say.} \quad . \quad . \quad (32)$$

Thus

$$\lambda=\kappa\left(\frac{\mathrm{V}^2l}{\mathrm{V}^2-a^2}-l\right).$$

Hence

$$\begin{aligned}(r^2-\mathrm{V}^2)\lambda&=\kappa\left\{\mathrm{V}^2\frac{r^2-\mathrm{V}^2}{\mathrm{V}^2-a^2}l-l(r^2-\mathrm{V}^2)\right\} \\ &=\kappa\left\{\mathrm{V}^2\left(\frac{r^2-\mathrm{V}^2}{\mathrm{V}^2-a^2}+1\right)l-lr^2\right\} \\ &=\kappa\left\{x\mathrm{V}-lr^2\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)\end{aligned}$$

Thus

$$(r^2-\mathrm{V}^2)(\lambda x+\mu y+\nu z)=\kappa\left\{\mathrm{V}(x^2+y^2+z^2)-r^2(lx+my+nz)\right\}=0,$$

$$\therefore \quad \lambda x+\mu y+\nu z=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Now x, y, z give the direction of the ray corresponding to the wave-normal l, m, n, and the direction of vibration λ, μ, ν. Thus the direction of vibration is normal to the ray.

Again, multiply (11) by λ/a^2 &c. and add. Then

$$\begin{aligned}&\frac{\lambda^2}{a^2}\left(\frac{\mathrm{V}^2}{a^2}-1\right)+\frac{\mu^2}{b^2}\left(\frac{\mathrm{V}^2}{b^2}-1\right)+\frac{\nu^2}{c^2}\left(\frac{\mathrm{V}^2}{c^2}-1\right) \\ &\quad=\frac{\mathrm{A}-\mathrm{B}}{\mathrm{B}}(l\lambda+m\mu+n\nu)\left(\frac{l\lambda}{a^2}+\frac{m\mu}{b^2}+\frac{n\nu}{c^2}\right) \\ &\quad=\frac{\mathrm{A}(\mathrm{A}-\mathrm{B})}{\mathrm{B}^2\mathrm{V}^2}(l\lambda+m\mu+n\nu)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)\end{aligned}$$

* Aldis, 'Tract on Double Refraction,' page 12.

Take the case in which A=0, and let

$$\lambda' = \lambda/a^2, \quad \mu' = \mu/b^2, \quad \nu' = \nu/c^2.$$

Then (35), (24), and (29) become

$$V^2 = a^2\lambda'^2 + b^2\mu'^2 + c^2\nu'^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

$$l\lambda' + m\mu' + n\nu' = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

$$\frac{l}{\lambda'}(b^2 - c^2) + \frac{m}{\mu'}(c^2 - a^2) + \frac{n}{\nu'}(a^2 - b^2) = 0. \quad . \quad (38)$$

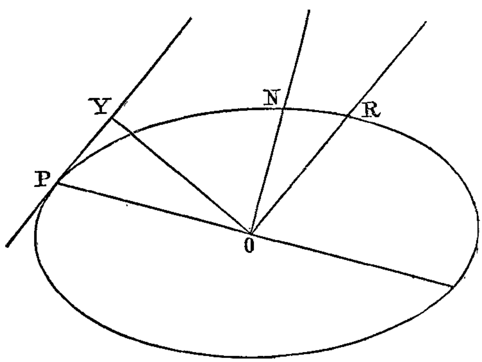
Thus, draw a plane normal to the direction of vibration to touch the ellipsoid. The quantities λ', μ', ν' will be the direction-cosines of the radius vector to the point of contact, and this radius vector, by (37), lies in the wave-front.

Moreover, the velocity of propagation is given by the length of the radius vector in the direction λ', μ', ν'; and this radius vector is an axis of the section of the ellipsoid by the wave-front. This is, of course, Fresnel's construction for the velocity. If Fresnel's construction were completely fulfilled, λ', μ', ν' would give the direction of vibration. As it is, that direction is given by $a^2\lambda'$, $b^2\mu'$, $c^2\nu'$; and these are the direction-cosines of the perpendicular on the tangent-plane to the ellipsoid at the point where it is met by λ', μ', ν'.

Moreover, this perpendicular clearly lies in the plane which contains the wave-normal, and the axis of the section of the ellipsoid by the wave-front; and since, according to Fresnel, the axis is the projection of the ray on the wave-front, the ray lies in this same plane.

Thus, in fig. 1, let O N represent the wave normal and O R

Fig. 1.



the ray. Take a section of the ellipse $a^2x^2 + b^2y^2 + c^2z^2 = 1$ by the plane N O R. Let O P be a radius of that section perpendicular to O N; O P is in the wave-front, and is the axis of the section of the ellipsoid by the wave-front. According to Fresnel, O P is the direction of vibration in the ray O R. According to the theory of the present paper this is not the case.

Let P Y be the tangent at P to the ellipse P N R, and O Y perpendicular on P Y. P Y is the trace on the plane P N R of the tangent-plane to the ellipsoid; and this plane is perpendicular to the plane P N R, so that O X is the normal to the tangent-plane at P. According to our theory, O Y is the direction of vibration, and moreover O Y is perpendicular to O R.

Now experiment* shows us that Fresnel's construction for the velocity is very closely indeed approached to; and hence A must be, if not actually zero, very small indeed. We have, however, no exact experimental evidence on the direction of vibration in a crystal; and it would be extremely difficult to devise an experiment which would decide between Fresnel's result and that of the theory now suggested. So far, then, as experimental evidence is concerned we may claim that the theory here given is in very close accordance with our present results. It has moreover the extreme advantage of basing the laws of double refraction on variations of the property of the æther, on which ordinary reflexion and refraction almost certainly depend.

Refraction occurs because the optical density of the æther is different in different media; double refraction, because in a crystal the optical density is different in different directions.

It remains now to consider what is meant by the optical density of the æther, and how it can vary in different media, or in different directions in the same medium. The phenomena of aberration and the other optical effects produced by the motion of transparent bodies are more easily explicable if we suppose the actual density of the æther as well as its rigidity to be the same in all bodies. Let us make this assumption for the present. Now the motion of the æther within a transparent body is not free; in addition to the forces arising from its own rigidity there must be others arising from the action of the transparent matter; and though we are ignorant of the nature of this action we can show, remembering that light-waves travel through the medium with a velocity which is independent of the amplitude, that the forces resolve themselves into two sets. One of these makes its appearance in such a way as to be equivalent to an increase in the density of the æther, while the other is

* Stokes, Proc. Roy. Soc.; B. A. Report, 1862. Glazebrook, Phil. Trans. 1879, part i.; 1879, part ii. Hastings, Silliman's Journal.

equivalent to an increase in its rigidity. Thus, suppose we have a magnetized steel spring vibrating, by placing it in water we increase the effective inertia of the spring, but by placing it in a magnetic field we may stiffen the spring. To express the same in analytical terms the solution of our differential equation is to be, supposing we have a wave travelling parallel to the axis of z,

$$u=k\sin n\,(z-\mathrm{V}t).$$

And this is a solution of

$$\frac{d^2}{dt^2}\left\{\rho+a+b\,\frac{d^2}{dz^2}+c\,\frac{d^4}{dz^4}+\ldots\right\}u$$
$$=\mathrm{B}\,\frac{d^2}{dz^2}\left\{1+a'\,\frac{d^2}{dz^2}+b'\frac{d^4}{dz^4}+\ldots\right\}u,$$

ρ being the density of the æther in free space, B its rigidity, and a, b, a', b', &c. constants. These terms or some of them may give the action of the matter on the æther, those in a, b, c &c. enter the equations as an effective increase of density, those in a', b', &c. as an increase of rigidity.

To state the same fact in another way, the equations of motion of the æther may be written

$$\rho\frac{d^2u}{dt^2}=(\mathrm{A}-\mathrm{B})\frac{d\delta}{dx}+\mathrm{B}\nabla^2u+\mathrm{X}, \quad . \quad . \quad . \quad (39)$$

where X represents the action of the matter on the æther. X is to have such a form as will allow the propagation of waves without the absorption of energy and with a velocity independent of the amplitude. It must also give us the ordinary laws of reflexion and refraction, and we must be able to explain by simple hypotheses the laws of double refraction, dispersion, anomalous dispersion, and metallic reflexion. Of late years a number of attempts have been made to find an expression for this quantity X. An account of them is given in my Report on Optical Theories (B. A. Report 1885). The most complete in some respects is that of Voigt, who, starting with the question as to what is the most general form consistent with the conditions imposed by the problem, comes to the conclusion that for an isotropic medium we may put

$$\mathrm{X}=-r\,\frac{d^2(u-\mathrm{U})}{dt^2}+a\,\frac{d^2(u-\mathrm{U})}{dz^2}+a'\frac{d^4(u-\mathrm{U})}{dz^2dt^2}-n(u-\mathrm{U}), \ (40)$$

where U is the displacement of the matter particles in the same element of volume as the æther, which has u, v, w for its component displacements. In a crystal other terms come in and the coefficients of these may be functions of the direction. The formulæ obtained by taking only the term $-n(u-\mathrm{U})$ have been discussed at length by Von Helmholtz* and Sir W. Thomson†.

Sir W. Thomson has shown that while the theory will account for dispersion it fails when we come to double refraction, for it makes that depend on the period‡.

The first term $-r\frac{d^2}{dt^2}(u-\mathrm{U})$ leads to equations for an isotropic medium which are practically identical with those employed by Ketteler. In fact he first suggested its use§.

He dismisses it shortly afterwards for reasons which do not seem to me to have great weight. The point is discussed in the Report on Optical Theories, p. 229. The theory leads to an account of dispersion which certainly agrees closely with experiment, and it will, as we shall see, explain double refraction satisfactorily, if we may assume Sir W. Thomson's theory of a contractile æther. For if we write ρ' for r in Ketteler's expression and ρ_0 for the density in free space, the equations of motion become

$$\rho_0\frac{d^2u}{dt^2}=(\mathrm{A}-\mathrm{B})\frac{d\delta}{dx}+\mathrm{B}\nabla^2u-\rho'\frac{d^2}{dt^2}(u-\mathrm{U}). \quad . \quad . \quad (41)$$

Now in a transparent medium in which there is no absorption, the value of U will be indefinitely small compared with that of u, and omitting it from the equation we get

$$(\rho_0+\rho')\frac{d^2u}{dt^2}=(\mathrm{A}-\mathrm{B})\frac{d\delta}{dx}+\mathrm{B}\nabla^2u. \quad . \quad . \quad . \quad . \quad (42)$$

This is the equation which we have been dealing with all along.

In a crystal the resistance offered to the motion of the æther will depend on the direction, and ρ' will have different values for the three axes. We then get

$$\rho_x\frac{d^2u}{dt^2}=(\mathrm{A}-\mathrm{B})\frac{d\delta}{dx}+\mathrm{B}\nabla^2u, \quad . \quad . \quad . \quad . \quad (43)$$

&c.,

and these are the equations for a crystal.

The assumption, therefore, that the mutual reaction depends on the relative accelerations of matter and æther gives us formulæ which explain double refraction. To explain dis-

* Helmholtz, Pogg. *Ann.* t. cliv. p. 582.

† Thomson, Baltimore Lectures.

‡ According to an account given recently in 'Nature' of the work of Prof. Lindemann on this subject, 'Nature,' August 23, 1888, he has surmounted this difficulty.

§ "Optische Controversen," Wied. *Ann.* t. xviii. p. 397, "Eine Dritte Annahme."

persion we adopt the view due originally to Sellmeier* and developed lately by Sir W. Thomson in the Baltimore lectures. According to this view it arises from the absorption of some of the light energy by the molecules of the body, owing to the fact that the period of the light-waves nearly synchronizes with that of the natural vibrations of the molecules. The term in U in our equation becomes appreciable. Of course in this case we need another equation to determine the motion of the matter molecules. The forces retarding this matter motion will arise partly from the reaction of the æther and partly from that of the matter itself. As to the expression for the latter we do not know what it is, but in order that the linear wave may be propagated it must depend on U and its differential coefficients. Let us suppose, taking the case of a plane wave travelling parallel to the axis of z, that the force is represented, as is assumed by Helmholtz, by

$$-\alpha^2\mathrm{U}-\gamma^2\frac{d\mathrm{U}}{dt}.$$

If differential coefficients of U with respect to z do come in, we can allow for them by supposing α^2 and γ^2 to be complex-operators. Such a supposition will alter the form of the expression for μ^2.

Now let us consider the æther and matter in a certain element of volume dv. Let ρ_0 be the density of the æther, and let us further suppose, for the present, that a portion only of the matter molecules in the element are disturbed; we shall have to deal with the average displacements of these matter molecules; let them be U, V, W, parallel to the axis; and let $\rho_1 dv$ denote the mass of matter within the element which is set in motion. If the whole of the matter in the element moves, ρ_1 will be the density of the matter; in general we may suppose it to be less than the matter density and to depend on the number of matter molecules set in motion by the light-waves.

Hence the equations become,

$$\left.\begin{aligned}\rho_0\frac{d^2u}{dt^2}+\rho'\frac{d^2}{dt^2}(u-\mathrm{U})&=(\mathrm{A}-\mathrm{B})\frac{d\delta}{dx}+\mathrm{B}\nabla^2u\\ \rho_1\frac{d^2\mathrm{U}}{dt^2}-\rho'\frac{d^2}{dt^2}(u-\mathrm{U})&=-\alpha^2\mathrm{U}-\gamma^2\frac{d\mathrm{U}}{dt}\end{aligned}\right\}\quad . \quad . \quad (44)$$

Thus putting $\rho_0+\rho'=\rho$, $\rho_1+\rho'=\rho_2$, so that ρ is the effective density of æther when loaded by the matter, ρ_2 the effective density of matter when loaded by æther, we have :—

* Sellmeier, Pogg. *Ann.* t. cxlv. pp. 399, 520; t. cxlvii. pp. 386, 525.

$$\left.\begin{aligned}\rho\frac{d^2u}{dt^2}-\rho'\frac{d^2U}{dt^2}&=(A-B)\frac{d\delta}{dx}+B\nabla^2u\\ \rho_2\frac{d^2U}{dt^2}-\rho'\frac{d^2u}{dt^2}&=-\alpha^2U-\gamma^2\frac{dU}{dt}\end{aligned}\right\} \quad . \quad . \quad (45)$$

The solution of equations practically the same as these has been given by Ketteler*. It will be useful to have it here for the sake of completeness, and also because the notation is different.

Let the solutions be given by

$$\left.\begin{aligned}u&=u_0\,e^{-kz+\iota n(z-Vt)/V}\,\dagger\\ U&=U_0\,e^{-kz+\iota n(z-Vt)/V}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

Then, on substituting, we have

$$\left.\begin{aligned}&u_0\left\{n^2\left(\rho-\frac{B}{V^2}\right)-B\left(\frac{2\iota nk}{V}-k^2\right)\right\}=U_0n^2\rho',\\ &U_0\{n^2\rho_2-\alpha^2+\gamma^2\iota n\}=u_0n^2\rho'.\end{aligned}\right\} \quad . \quad . \quad (47)$$

Thus

$$\left\{n^2\left(\rho-\frac{B}{V^2}\right)-B\left(\frac{2\iota nh}{V}-k^2\right)\right\}\{n^2\rho_2-\alpha^2+\gamma^2\iota n\}=n^4\rho'^2; \quad (48)$$

$$\therefore\left\{n^2\left(\rho-\frac{B}{V^2}\right)+Bk^2\right\}(n^2\rho_2-\alpha^2)+\frac{2B\gamma^2n^2k}{V}=n^4\rho'^2; \quad (49)$$

$$\gamma^2\left\{n^2\left(\rho-\frac{B}{V^2}\right)+Bk^2\right\}-\frac{2Bk}{V}(n^2\rho_2-\alpha^2)=0 \quad . \quad . \quad . \quad (50)$$

$$\therefore\frac{1}{V^2}-\frac{k^2}{n^2}=\frac{\rho}{B}-\frac{2(n^2\rho_2-\alpha^2)k}{\gamma^2n^2V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

and

$$\frac{2Bk}{V\gamma^2}\{(n^2\rho_2-\alpha^2)^2+n^2\gamma^4\}=n^4\rho'^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

Thus

$$\frac{2k}{nV}=\frac{\gamma^2\rho'^2n^3}{[\{n^2\rho_2-\alpha^2\}^2+\gamma^4n^2]B} \quad . \quad . \quad . \quad . \quad . \quad (53)$$

$$\frac{1}{V^2}-\frac{k^2}{n^2}=\frac{\rho}{B}-\frac{\rho'^2n^2(n^2\rho_2-\alpha^2)}{[(n^2\rho_2-\alpha^2)^2+n^2\gamma^4]B} \quad . \quad . \quad (54)$$

Let

$$\alpha^2=\nu^2\rho_2.$$

Then

$$\frac{2k}{nV}=\frac{\gamma^2\rho'^2n^3}{[\rho_2(n^2-\nu^2)+\gamma^4n^2]B} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

$$\frac{1}{V^2}-\frac{k^2}{n^2}=\frac{\rho}{B}-\frac{\rho'^2\rho_2n^2(n^2-\nu^2)}{[\rho_2^2(n^2-\nu^2)^2+\gamma^4n^2]B} \quad . \quad . \quad (56)$$

* Ketteler, *Theoretische Optik*, § 42, and various papers alrady referred to.

† The symbols k, n, λ, μ, ν, have no longer the same signification as above, pp. 524 *seq.*

Also we find

$$\frac{U_0}{u_0}=\frac{n^2\rho'}{\rho_2(n^2-\nu^2)+\gamma^2\iota n}=Re^{-\iota\theta}, \quad . \quad (57)$$

where

$$\left.\begin{aligned} R^2&=\frac{n^4\rho'^2}{(n^2-\nu^2)^2\rho_2^{\,2}+\gamma^4n^2} \\ \text{and} \qquad \tan\theta&=\frac{\gamma^2 n}{(n^2-\nu^2)\rho_2} \end{aligned}\right\} . \quad . \quad . \quad . \quad . \quad (58)$$

Now we notice that k is a quantity which depends on γ^2 and is small therefore when γ^2 is small. In a transparent body k must be practically zero, and hence we infer that for a transparent body γ^2 is extremely small. The only reason for retaining the term at all lies in the fact that if we put $\gamma^2=0$, then for the value $n^2=\nu^2$ we have the ratio U_0/u_0 infinite.

Taking, then, a case in which γ^2 is zero, and remembering that our solution fails for the critical value ν^2 of n^2, we have in general U_0/u_0 small, and

$$\frac{1}{V^2}=\frac{\rho}{B}-\frac{\rho'^2n^2}{B\rho_2(n^2-\nu^2)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

Now let V_0 be the velocity *in vacuo* of light of the frequency n ; λ its wave-length.

Then

$$n=\frac{2\pi V_0}{\lambda}.$$

Put

$$\nu=\frac{2\pi V_0}{\lambda_1},$$

and substitute in (59)

$$\frac{1}{V^2}=\frac{\rho}{B}+\frac{\rho'^2}{B\rho_2}\frac{\lambda_1^{\,2}}{\lambda^2-\lambda_1^{\,2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

Also, if $\mu=V_0/V=$ the refractive index. Since $V_0^{\,2}=B/\rho_0$,

$$\mu^2=\frac{\rho}{\rho_0}+\frac{\rho'^2}{\rho_0\rho_2}\frac{\lambda_1^{\,2}}{\lambda^2-\lambda_1^{\,2}}. \quad . \quad . \quad . \quad . \quad . \quad (61)$$

The quantity ρ/ρ_0 is the square of the refractive index for waves of infinite length ; put it $\mu_\infty^{\,2}$, and write C for $\rho'^2/\rho_0\rho_2$. Then

$$\mu^2=\mu_\infty^{\,2}+\frac{C\lambda_1^{\,2}}{\lambda^2-\lambda_1^{\,2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

This is Ketteler's dispersion formula, which he has proved agrees well with the results of experiment over a long range of values of λ*.

By supposing, as is done by Sir W. Thomson in the Balti-

* Ketteler, Wied. *Ann.* xii. pp. 363, 481, xv. p. 336, and elsewhere.

more Lectures, that there are a number of possible periods of motion for the matter particles corresponding to a series of values for ν^2 or $\lambda_1{}^2$, we get a series of terms in the expression for μ^2, which becomes

$$\mu^2=\mu^2_\infty+\frac{C\lambda_1{}^2}{\lambda^2-\lambda_1{}^2}+\frac{D\lambda_2{}^2}{\lambda^2-\lambda_2{}^2}+\quad . \quad . \quad . \quad . \qquad (63)$$

Ketteler has shown that for Iceland spar, taking Mascart's measurements of refractive index, the above formula with three terms agrees very closely with experiment from

$$\lambda=\cdot76013 \text{ to } \lambda=\cdot31775.$$

The greatest difference is ·0001 in the value of the refractive index.

This last formula may be transformed to a more useful form if we suppose λ_1 is large compared with λ, so that we may neglect $(\lambda/\lambda_1)^4$ and higher powers. We get then

$$\mu^2=\mu^2_\infty-C\left(1-\frac{\lambda^2}{\lambda_1{}^2}\right)^{-1}+\frac{D\lambda_2{}^2}{\lambda^2-\lambda_2{}^2}$$

$$=\mu^2_\infty-C-\frac{C}{\lambda_1{}^2}\lambda^2+\frac{D\lambda_2{}^2}{\lambda^2-\lambda_2{}^2}$$

$$=a^2-k^2\lambda^2+\frac{D\lambda_2{}^2}{\lambda^2-\lambda_2{}^2}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (64)$$

Again, according to Ketteler * this formula will give the dispersion in quartz with considerable accuracy from $\lambda=2\cdot14$ to $\lambda=\cdot18$, or through about 12 octaves; while it agrees very fairly with Langley's † observations for flint glass from

$$\lambda=2\cdot356 \text{ to } \lambda=\cdot3440.$$

For flint glass the values for the constants given by Ketteler are

$$k^2=\ \cdot009076 \qquad D=\cdot60714$$

$$a^2=2\cdot44137 \qquad \lambda_2=\cdot029929$$

According to the dispersion formula given above, the value of μ^2 is infinite for $\lambda=\lambda_1$ or $\lambda=\lambda_2$. For these values the light will be absorbed in the medium. Moreover, for values of λ somewhat greater than λ_1, μ^2 becomes a real negative quantity, and the light ceases to be transmitted. Now the characteristic property of substances which show metallic refraction is, according to Jamin and Quincke ‡, that μ^2 is a real negative quantity.

Thus the theory will explain ordinary metallic reflexion by

* Ketteler, Wied. *Ann.* xxx. p. 312.

† Langley, "Professional Papers of the Signal Service," No. xv. A Report of the Mount Whitney Expedition, p. 226.

‡ See Hon. J. W. Strutt [Lord Rayleigh], "Reflexion of Light from Intensely Opaque Matter." Phil. Mag. 1872.

the supposition that λ_1 is less than the wave-length of any part of the visible spectrum. Now λ_1 is the wave-length corresponding to the free periods of the matter vibrations. Thus the free periods for the matter vibrations are less than those for any visible light. The application of the theory to thin metallic films, and to the small prisms investigated by Kundt, requires further consideration, and must be left for the present.

If this explanation be true, then a substance which is opaque to light might be transparent to waves of greater length, all that is required is that the length of such waves should be greater than the critical length λ_1. Professor J. J. Thomson has recently found this to be the case for ebonite, which transmits easily the long waves of electric disturbance in experiments such as those of Hertz. Thus somewhere between these electric vibrations and those of light, ebonite has a band of strong absorption, and, moreover, there are no free periods possible for the free-matter vibrations in ebonite, which are less than the periods of the other vibrations.

Thus, to explain the effect of a metallic medium there is no need to invoke the aid of the terms in γ^2 in the equations of motion. Part of the effect may, it is true, be due to the existence of such terms; it is sufficient, however, that λ_1 should be somewhat greater than λ, then μ^2 will, for some values of λ, be less than unity, and for others a real negative quantity. The appendix to Sir W. Thomson's Baltimore Lectures contains a discussion of this point, and the formulæ there given become those of the theory now considered if we write for C_1 &c. in Thomson's equation $-4\pi^2 C_1/\tau^2$. (See 'Report on Double Refraction,' p. 245 *et seq.*)

The theory will, without serious modification, give us the formula originally due to Fresnel and now fully verified by the experiments of Fizeau and Michelson, connecting the velocity of light in a moving medium with the velocity of the medium.

For suppose that the æther is at rest, and that a transparent body is moving through it with velocities L, M, N, parallel to the axes. Then in estimating the relative accelerations of the æther and the matter at a point fixed in the body, we must remember that the value of $\frac{d}{dt}$ will be

$$\frac{d}{dt}+\mathrm{L}\frac{d}{dx}+\mathrm{M}\frac{d}{dy}+\mathrm{N}\frac{d}{dz}.$$

So that taking the case of motion in the direction of propagation, the term $\rho'\frac{d^2u}{dt^2}$ becomes $\rho'\left(\frac{d}{dt}+\mathrm{N}\frac{d}{dz}\right)^2 u$; and this, if we neglect N^2 as small compared with the other quantities, gives us as the equation of motion,

$$\rho \frac{d^2u}{dt^2} + 2N\rho' \frac{d^2u}{dzdt} = B \frac{d^2u}{dz^2}. \quad . \quad . \quad . \quad (65)$$

Hence if $u = u_0 \sin \frac{2\pi}{\lambda}(z - Vt)$,

$$\rho V^2 - 2N\rho' V - B = 0.$$

Hence

$$V = \frac{N\rho' \pm \sqrt{N^2\rho'^2 + B^2}}{\rho} = \frac{B + N\rho'}{\rho}.$$

$$\therefore \quad V = V_0 + \frac{N\rho'}{\rho}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

to the same approximation, if V_0 is the velocity when the medium is at rest.

Again,

$$\mu^2 = \frac{\rho}{\rho_0} = \frac{\rho_0 + \rho'}{\rho_0},$$

$$\therefore \quad \frac{\rho'}{\rho_0} = \mu^2 - 1 ;$$

and

$$\frac{\rho'}{\rho} = \frac{\mu^2 - 1}{\mu^2},$$

$$\therefore \quad V = V_0 + \frac{\mu^2 - 1}{\mu^2} N. \quad . \quad . \quad . \quad (67)$$

And this is Fresnel's formula, which has been obtained by Boussinesq in a similar manner.

The consideration of phenomena connected with the rotation of the plane of polarization must be deferred to a future article.

It remains now to refer to one point of great importance which the theory as it stands will not explain.

Experiment shows us that in the case of the reflexion of light from transparent media Fresnel's tangent-formula does not hold. Some light is reflected near the polarizing angle. According to the theory the tangent-law is true at least when A vanishes.

Now it is clearly true that there must be a thin layer of the æther near the separating surface of two media, air and glass say, across which the optical density of the æther changes from that of air to that of glass. If this layer be infinitely thin compared with the wave-length, then the transition is practically sudden; but if the layer has a thickness comparable with the wave-length, then effects such as are actually observed would occur. And L. Lorenz * has shown that the effects of elliptic polarization observed by Jamin would agree numerically with the results of a theory of gradual transition, if the thickness of the variable film lies between $\frac{1}{10}$ and $\frac{1}{100}$ of a wave-length.

* Pogg. *Ann.* t. cxiv. p. 238; Glazebrook, Report on Optics, p. 188.

To this explanation the objection * has been made that the phenomena resemble those shown by thin films, and that the reflected light ought to be coloured. We may reply to this that the film is comparable with the thickness of the black film in Newton's rings, so that the colour shown, if any, would be that of the blue of the first order, and would probably —the light being very faint—hardly be noticed as colour by an observer who was not specially directing his attention to that point. I hope, however, shortly to investigate this question by direct experiment. Such observations of a preliminary character as I have made have shown a bluish tint in the reflected light. Moreover, if this light appear blue, so, too, ought the light which is reflected in considerable quantity from the black spot in a soap-film. In fact, if we take Reinold and Rücker's value for the thickness of the black spot in a soap-film, the quantity of light reflected from it is much greater than that observed by Jamin near the polarizing-angle. For, taking the case of normal incidence, we have for the intensity of the reflected light in terms of the incident the value

$$I=\frac{4\left(\frac{\mu-1}{\mu+1}\right)^2\frac{4\pi^2D^2}{\lambda^2}}{\left\{1-\left(\frac{\mu-1}{\mu+1}\right)^2\right\}^2}.$$

If we put $\mu=\frac{4}{3}$, $\qquad \frac{\mu-1}{\mu+1}=\frac{1}{7}.$

Then, approximately, $I=\frac{16\pi^2}{49}\times\frac{D^2}{\lambda^2}=\frac{10}{3}\frac{D^2}{\lambda^2}.$

Now, according to Reinold and Rücker,

$$D=\frac{1}{50}\lambda;$$

$$\therefore\quad I=\frac{10}{3\times 2500}=\cdot 0013.$$

Thus over one thousandth part of the incident light ought theoretically to be reflected, and the colour of this ought to be mixed in the inverse ratio of the square of the wave-lengths. I am not aware that any careful photometric observations on this light have been made; at any rate it is, I think, nowhere stated that it shows colour.

Now as to the Jamin effect †, we have

$$I=M^2\tan^2(\phi-\phi')$$

* Hon. J. W. Strutt [Lord Rayleigh], "On the Reflexion of Light from Transparent Matter," Phil. Mag. 1871.

† Jamin, *Annales de Chimie et de Physique*, 3 sér. t. xxix. *Cours de Physique*, iii. p. 525.

at the polarizing-angle, and, in Jamin's notation,

$$M = \epsilon \sin\phi.$$

According to his observations for flint glass,

$$\epsilon = \cdot 017, \quad \phi = 59^\circ\cdot 44.$$

Whence

$$I = \cdot 000069.$$

If we take Quincke's * observations, we find, from his table for reflexion at the surface of flint glass, the value $I = \cdot 000096$. Thus the value of I for glass is less than ·0001, or one tenth the amount of light that is reflected from the black film of a soap-bubble. Hence the light reflected from the bubble being brighter ought to show more colour than the light reflected from glass near the polarizing-angle.

Two explanations may be given of the fact that colour has not been noticed—except, possibly, in the rough observation of my own already referred to—in either case. The one is that, owing to its faintness, it has escaped the notice of observers who were not specially looking for it; the other lies in the fact that there seems some reason to suppose that our eyes are sensitive to light before they appreciate distinctions of colour. Either of these would, I think, be sufficient to account for the facts, and would allow us to believe that the explanation of the Jamin-effect given by Lorenz is the true one, though possibly further experiments, which I hope shortly to undertake, may be necessary to prove this. This elliptic polarization depends greatly, it is true, as Wernicke † has shown lately, on the nature of the surface and of the means taken to polish it; but he comes to the conclusion "that it is a general property of bodies modified by the presence of a surface-film, but not entirely explained by that."

This surface-film of Wernicke's is, however, quite different from the surface-layer of æther of variable density considered by Lorenz. Wernicke's film is caused by the presence of foreign matter; the layer of varying density is necessary to the transition from air to glass, and it is only a question of how thick this layer is in comparison with the wave-length.

We conclude, then, that the theory here put forward accounts satisfactorily for reflexion and refraction both by transparent bodies and by metals, also for double refraction and dispersion, including the anomalous dispersion of such substances as cyanin, fuchsin, or the other anilin dyes, while it leads, in addition, to the correct expression for the velocity of light in a moving medium. As already stated, I hope to treat, in another paper, the consideration of the properties of quartz, the rotation produced by sugar, and the rotation in a magnetic field.

* "Optische Experimental Untersuchungen," Pogg. *Ann.* Band 127, 128, &c.

† Wied. *Ann.* xxx. p. 468.