as it were, at random, it cannot all be accident. There are, however, exceptions: we find them in the groups

	At. Weight.	At. Vol.	Melting Point.
Manganese,	27·6	44	Highest *heat* of wind furnace.
Iron,	28	44	" "
Cobalt,	29	44	" "
Nickel,	29	44	" "
Copper,	32	44	1996° F.
Phosphorus,	33	211	111°
Antimony,	129	224	Red heat.
Bismuth,	213	270	507°

Manganese and iron, and perhaps cobalt and nickel, follow this law, but copper varies very much; for this we can see no reason. Phosphorus and antimony follow the law, but bismuth comes between. What can influence it? Look at its atomic volume; it differs 59 from that of phosphorus. We cannot, therefore, be much surprised at its having a different melting point.

These facts support Mr. Coleman's views. The subject is interesting and well worth discussing.

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*Mathematical Theory of the Dynamical Effects of Heat given to a Permanent Gas.** By M. J. Bourget, Professor of Mathematics in the Faculty of Sciences at Clermont.

Up to the present time, the theory of the motive power of heat has been treated, by assuming *â priori* the following propositions:—

It is absurd to admit the possibility of creating either moving force or heat.

Heat cannot be made to pass from a colder to a warmer body.

In all cases where mechanical work is produced by heat, there is a consumption of a quantity of heat proportional to the work done; reciprocally, this quantity of heat may be represented by a quantity of mechanical work equal to that before spoken of.

I am about to undertake the same subject in a different way, confining myself to the case in which a permanent gas is the vehicle of the heat.

It seemed to me that if the principles above mentioned are true; if it be true that heat and mechanical work may be regarded as homo-

*Works to be consulted on the question of the Mechanical Equivalent of Heat:—

Joule.— Memoir on the heating effects of magneto-electric currents; and on the mechanical equivalent of heat. *Annales de Chimie et de Physique,* Tome XXXIV.

Joule.— On the mechanical equivalent of heat.

Mr. W. Thomson.— Examination of Carnot's theory of the motive force of heat.

Clausius.— On the motive force of heat. *Annales de Chimie et de Physique,* Tome XXXV.

Quintus Icilius.— Memoir on the numerical values of the constants which enter into the expression for the heat disengaged by currents.— *Annales de Chimie et de Physique,* Tome LI.

Clausius.— On the motive force of heat, and on the laws resulting from it.— *Bibliotheque Universelle de Geneve,* Tome XXXVI.

Mr. W. Thomson.— Two memoirs on the dynamic theory of heat.— *Liouville's Mathematical Journal,* Tome XVII.

Reech.— General Theory of the dynamical effects of heat.— *Journal des Mathematics,* Tome XVIII.

See also the memoir entitled "A new system of Air Engine, deduced from a comparison of the systems of MM. Ericsson and Lemoine," by M. Reech.

geneous, and capable of being transferred the one into the other by equivalents; if, in one word, perpetual motion is impossible for hot-air engines as for others; this ought to be demonstrated, starting from known physical laws, and the formulæ deduced from these laws. I have not been deceived in my hope; and the mechanical equivalent of heat will here be seen deduced from the laws of Mariotte, Gay-Lussac, Dulong, and Regnault.

In another essay, in collaboration with M. Burdin, I studied the air machines, using the formulæ of Poisson which connect the pressures, densities, and temperatures of a given mass of gas, compressed or dilated, without being directly heated or cooled. Many of the consequences here noticed flowed from these formulæ. But it was not without hesitation that I assumed the formulæ of this illustrious mathematician as my base; the reasoning by which they were reached does not appear to me free from objection.

I have happily succeeded in removing this difficulty, and I have found a new and very simple demonstration of the same formulæ, which clearly shows the only hypothesis necessary to arrive at them.

Although confined to permanent gases, my analysis appears to me to give a certain degree of probability to that fruitful and seductive conception, the homogeneousness of the natural agents. Is it not, in fact, a thing well worthy of attention, that laws and formulæ found without any pre-occupation with the new principles, implicitly contain them?

SECTION 1.—Definitions and Preliminary Formulæ.

1. *Mathematical Representation of the State of a Gas.*

Three quantities determine completely the state of a permanent gas: its elastic force (p); its volume (v); and its temperature (t). We shall express its pressure in kilogrammes per square metre;* its volume will be referred to the cubic metre, and its temperature measured in centigrade degrees. To fix the ideas, we may, in what follows, suppose the gas enclosed in a cylinder of a square metre cross section, beneath a movable piston without weight, and always loaded with a pressure equal to its own elastic force.

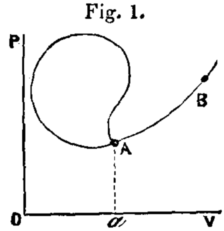
Between these three variables, p, v, and t, there is a relation resulting from the laws of Mariotte and Gay-Lussac combined; for by denoting by p_0, v_0, and t_0 the values of these variables for another state of the gas, we have, $\frac{p\,v}{1+a\,t}=\frac{p_0\,v_0}{1+a\,t_0}$; whence, $pv=m(1+at)$ (1)

m being a constant, and a the co-efficient of dilatation, which is sensibly constant for all the gases. To determine m, make $t_0=0°$, call H the normal pressure of 0·76 referred to the square metre, and let us suppose that we are considering one cubic metre of gas at this pressure and temperature; then we shall have $m = \text{H} = 10{,}333$ kil.; whence we obtain, $p\,v = \text{H}\,(1+a\,t)$. (2)

*The French units have been preserved in this translation. The kilogramme is 2·2047 lbs.; the cubic metre is 35·352 cub. ft., or 1·331 cub. yds. The square metre is 1·196333 sq. yds., or 10·767 sq. ft. The centigrade degree is equal to 1·8 Fahrenheit degrees. The millimetre is 0·039354 ins.; whence, 0·76 mil. = 29·93184 ins. The normal pressure used in England and in this country is 30 inches.

This is the form which we shall use in the subsequent calculations.

Fig. 1.

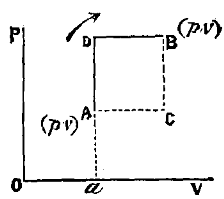


According to equation (2), it is sufficient to know the two quantities p and v, in order to know the state of a gas. Let us then trace two rectangular axes, and let us take the volumes as the abscissas and the pressures as ordinates. It is evident that any state of the gas will be represented by a point such as A; a series of different states infinitely near each other will form a curve such as A B; if this curve re-enters itself and returns to its point of departure, it will form a *closed circuit.*

If we assume $p\,v = \text{H}\,(1 + \alpha\,t) = \text{const.} = \theta,$ (3)
we obtain a hyperbola having for asymptotes the axes of co-ordinates; any point in this curve represents a state of the gas for which the temperature is constant.

2. *Expenditure of Heat in describing a rectangular closed circuit.*

Fig. 2.



Let A $(p\,v)$ be the initial state of a gas; by lines parallel to the axes, trace the closed circuit, A D B C A; cause the gas to pass through the different states which correspond to this circuit described in the direction of the arrow, and let us seek for the heat expended.

1. While the gas is passing from A to D, it is being heated under constant volume; the temperature t_1 is given by the formula $p'\,v = \text{H}\,(1 + a\,t_1)$, which, combined with $p\,v = \text{H}\,(1 + a t)$, gives $(p' - p)\,v = \text{H}\,a\,(t_1 - t)$.

The expenditure of heat will be $q = \text{D}\,c'\,(t' - t)$,
D denoting the weight of a cubic metre of air at 0°, and c' the specific heat under constant volume.

By substitution, this expression becomes $q = \dfrac{\text{D}}{\text{H}\,a}\,c'\,v\,(p' - p).$ (4)

2. While the gas passes from the state D to the state B, it is being heated under constant pressure; the temperature t' is given by the formula $p'\,v' = \text{H}\,(1 + a\,t')$, which combined with $p'\,v = \text{H}\,(1 + a\,t_1)$, gives $p'\,(v' - v) = \text{H}\,a\,(t' - t)$; the expenditure of heat is $q_1 = \text{D}\,c\,(t' - t)$, c being the specific heat under constant pressure.

This formula becomes by substitution, $q_1 = \dfrac{\text{D}}{\text{H}\,a}\,c\,p'\,(v' - v).$ (5)

3. While the gas passes from the state B to the state C, it is cooled under constant volume; its temperature t_2 is given by the formula $p\,v' = \text{H}\,(1 + a\,t_2)$, which combined with $p'\,v' = \text{H}\,(1 + a\,t')$, gives $(p' - p)\,v' = \text{H}\,a\,(t' - t_2)$. The heat obtained will be $q' = \text{D}\,c'\,(t' - t_2)$, or

$$q' = \frac{\text{D}}{\text{H}\,a}\,c'\,v'\,(p' - p). \qquad (6)$$

4. Finally, while the gas passes from the state C to its primitive

state A, it cools under constant pressure; its temperature again becomes t given by the formula $p\,v = \text{H}\,(1 + a\,t)$, which combined with $p\,v' = \text{H}\,a\,(1 + a\,t_2)$, gives $p\,(v' - v) = \text{H}\,a\,(t_2 - t)$.

The heat obtained will be $q'_1 = \text{D}\,c\,(t_2 - t)$ or by substitution, $q'_1 = \frac{\text{D}}{\text{H}\,a}\,c\,p\,(v' - v)$. (7)

In fine the expenditure of heat will be

$$\text{Q} = q + q_1 = \frac{\text{D}}{\text{H}\,a}\,[c'\,v\,(p' - p) + c\,p'\,(v' - v)]$$

the heat obtained will be

$$\text{Q}' = q' + q'_1 = \frac{\text{D}}{\text{H}\,a}\,[c'\,v'\,(p' - p) + c\,p\,(v' - v)]$$

The whole expenditure will therefore be

$$\text{Q} - \text{Q}' = \frac{\text{D}}{\text{H}\,a}(c - c')\,(p' - p)\,(v' - v)$$

As this quantity cannot be equal to 0, we reach this remarkable result:

When a gas leaves any given state, it cannot return to it, after following a rectangular circuit of successive states, without the disappearance of a quantity of heat which is proportional to the surface $(p' - p)\,(v' - v)$ of the circuit.

3. *Remark* 1*st.*—The preceding law proves that the quantity of heat necessary to pass a gas from one state A, to another B, depends not only on these extreme states, but also on the intermediate ones. It is, I believe, the first time that this truth has been shown by deducing it from the admitted laws of the action of heat upon the gases. In fact, I find in a condensation of the work of Clausius, by my friend M. Verdet (*Annales de Chimie et de Physique*, tome xxxv, p. 483), the following passages:—"When a gas passes from the temperature t_0 and volume v_0, to the temperature t and volume v, it is generally admitted that the heat which it gains or loses depends only on the initial and final values of the temperature and volume. This is impossible if there be an equivalence between the mechanical work and the heat."

We see from the above process that it is not necessary to invoke any new principle for the purpose of proving this impossibility.

4. *Remark* 2*d.*—We might have followed the circuit in the opposite direction; there would then have been *gain* instead of *loss* of heat, and the quantity gained would still have been proportional to the area A C B D.

5. *Remark* 3*d.*—Our reasoning shows that $a\,c\,c'$ depend neither on the temperature nor on the pressure. M. Regnault has established that a and c are subject to this law. As to c' its determination is more difficult; MM. Gay-Lussac and Welter deduced from their experiments, that the ratio (γ) of these two specific heats is sensibly independent of the temperature and pressure, in the case of the atmospheric air; and it follows that c' must be nearly invariable also.*

* At the same time, we do not think that much confidence should be given to this result. We shall therefore make no supposition as to c', but shall place ourselves in such limits of temperature and pressure, as to allow c' to be considered constant as well as a and c.

6. *Remark 4th.*—If the point B is indefinitely approximated to A, the expenditure of heat approaches zero. In other words, to pass from the state A to another B infinitely near it, the same amount of heat must be expended whether you follow the route A c B or A D B. The two expenditures are, omitting an infinitely small quantity of the second order,

$$Q = \frac{D}{H\,a}\,(c'\,v\,d\,p + c\,p\,d\,v).$$

7. *Effective work of the gas in describing the rectangular circuit.* Supposing, as we have done, the gas to be in a cylinder of a square metre of cross-section, and under a piston always equilibriated. It is seen that the effective work along the path A B D is $W = p'\,(v' - v)$.

The resistance through the path B c A is $W' = p\,(v' - v)$.

Then the effective work of the gas through the whole of the closed circuit is $W - W' = (p' - p)\,(v' - v)$

and is therefore represented by the area of the rectangle formed by the circuit.

8. *Elementary Demonstration of the Principle of the Equivalent.* If we compare equations (10) and (14) we deduce

$$Q - Q' = \frac{D\,(c - c')}{H\,a}\,(W - W').$$

The co-efficient under the vinculum is constant for the same gas, for the pressures and temperatures used. Let us make then $E = \frac{H\,a}{D\,(c - c')}$

and we shall have $Q - Q' = \frac{1}{E}\,(W - W').$

Therefore, "*The quantity of heat lost is proportional to the effective work done, and reciprocally; so that to each calory* lost, corresponds an effective work produced equal to* $E^{k\,m.}$.

Therefore without disquieting ourselves to know what heat is, we may at all events say, *that every thing takes place as though heat transformed itself into mechanical force at the rate of* $E^{k\,m.}$ *for each calory lost.*

This number E may be called *the mechanical equivalent of heat;* assuming $H = 10{\cdot}333$ k; $a = 0{\cdot}003665$; $D = 1{\cdot}293187$; $c = 0{\cdot}2377$; $\gamma = \frac{c}{c'} = 1{\cdot}41$ (Masson). We find in round numbers $E = 424$ kilogrammetres.†

9. *Remark 1st.*—This number necessarily presents some uncertainty, since physicists are not all agreed as to the value of γ. This is the reason that we have only assumed only the whole numbers.

10. *Remark 2d.*—If the closed circuit had been described in the opposite direction, the work would have been resisting in place of effective; nevertheless the formula would not have changed and we should have reached the following conclusion:

*A *calory* is an unit of heat; as for instance the quantity of heat necessary to raise 1 kilogramme of a gas one degree of temperature.

†This, in English measure, is 773 lbs. raised one foot high per degree Fahrenheit.

"*The heat produced is proportional to the resistance of the gas in the proportion of* E$^{\text{k m.}}$ *for each calory.*"

The first proposition includes both cases, provided we admit once for all, that a negative quantity of heat lost is that quantity of heat gained; and that a negative quantity of work done represents that quantity of resistance.

11. *Remark* 3*d*.—The demonstration which we have given of the transformation of heat into mechanical work, in this particular case, may be introduced into the elements of Physics; it moreover shows the method to be followed to verify experimentally the truth of all our considerations.

In fact: either *first* there is an equivalent of heat, and in that case the number E is mathematically constant; then for all the gases we shall have

$$\frac{a}{\mathrm{D}\,(c - c')} = \text{const.} = \frac{a}{\mathrm{D}\,c\left(1 - \frac{1}{\gamma}\right)};$$

and this law is mathematical. Besides, as a is sensibly the same for all we must have $\mathrm{D}\,(c - c') = \mathrm{D}\,c\left(1 - \frac{1}{\gamma}\right) = \text{const.}$

Therefore: *in different gases, the differences of the two specific heats for the same weights, are inversely as their densities:* or taking the specific heats of equal volumes; *the difference of these specific heats is the same for all the gases.*

Moreover, Dulong proposed this law that equal volumes of different gases require the same quantity of heat to raise them the same number of degrees; this is the law of the specific heat of atoms. Therefore $\mathrm{D}\,c$ = constant for a certain number of permanent gases; therefore for all these γ = constant.

For the same gas, the absolute invariability of E requires the invariability of $c - c'$, and consequently that of c', since c is nearly invariable; and consequently that of γ.*

Or else, *secondly*, the equivalent E does not exist in the sense assumed by Clausius, Joule, Seguin, &c. In this case our formula will still remain for small intervals of temperature and pressures; but for every different interval the co-efficient E is different. In this case E = function (*p. t.*), and the preceding conclusions do not stand.

Thus we see that if we should succeed in proving that the ratio γ is constant for the same gas, we should have made an important verification of the existence of a mechanical equivalent of heat.

Our calculations show plainly that this hypothesis of the invariability of γ necessarily leads to the admission of the transformation of heat into mechanical work. This is the reason that this may be deduced from the formulæ of Poisson, which only assume this invariability.

*All these results were found by Clausius, as consequences of the hypothesis made *a priori* that there was a mechanical equivalent of heat.